COMMUNICATING LIKE A MATHEMATICIAN

What's the issue?

Students' working is often peppered with incorrect usage of symbols and very little written explanation of methodology. When read literally, it can make no sense and leaves the teacher deciding whether or not they should impute understanding. Does the student lack conceptual understanding, or do they simply have a poor grasp of mathematical language? Maybe both?

Added to this, many students struggle to read mathematical statements out loud, in part because they aren't familiar with the names of the symbols being used, and don't know how to say the mathematical syntax. For example, x^2 may be spoken as "x two".

There is significant evidence that many students do not properly grasp the language of mathematics, which makes learning the subject that much harder for them!

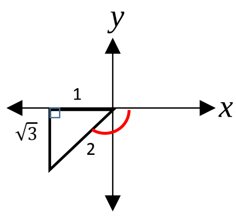
Mishaps? Perhaps not...

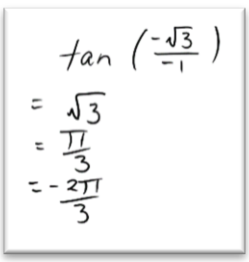
Here is a lament of one University first-year mathematics lecturer...

> *" They just don't write down what they're doing, they don't explain. It's just literally, they think they just need to write a page of equations with each [of] these funny little symbols joining everything together and they'll think they're done."*

Explaining workings and reasoning in plain English is an important part of communicating mathematically, and students often struggle with it. They sometimes find using symbols correctly challenging, with the most common error being the equals sign. Of 27 senior secondary teachers and first year university educators that we spoke with in open-ended interviews, 41% were concerned that their students don't use the equals (=) sign correctly, often appearing to use it to mark the next step of thinking.

In one example of this a student who was asked to find the angle in red in the diagram at the right found the 'correct' number for their answer in their final line, but their workings are nonsense, if read literally.





Mishaps in school maths...

Click below to see some more specific examples of student errors at different year levels, which can highlight there's a problem with communication.

- *Clues in Years 7-9*
- *Clues in Years 10-12*

Ideas from the classroom…

You don't have to use symbols *all* the time in maths, but when you do, they have to be right.

Here are some ideas to try at all levels in secondary school, to help your students become master-level mathematical communicators:

- Check what you've written on the whiteboard to be sure it makes sense if someone else reads it independently of conversation/explanation.
- Keep the equals sign underneath each other & remind students that whenever an equals sign is used, it's like a set of scales and each side must balance.

- Encourage students to read out their problems for you to transcribe at the whiteboard. Speaking could help to consolidate their writing skills in maths and vice-versa.
- Encourage students to share their workings with a partner to read – do they make sense?
- Encourage students to include an abbreviated commentary throughout their workings. This should include concepts like defining variables, which theorems are being used and important logical steps being undertaken. One approach to help novices structure this is to use a two-column approach, where explanations are contained to the right of each key step.
- Reduce the number of multiple-choice questions in homework & tests. Only the end result is valued in this type of question, which sends a message that mathematics is focused on the final answer.
- Attribute marks to correct use of notation in tests and assignments. They'll thank you in the long-run!
- Have students keep a word & symbol glossary: add new symbols to it when encountered, including the commonly used Greek letters.
- Put up posters of the Greek alphabet in the maths rooms. There are many YouTube videos to help too, for example, this Greek Alphabet Song.

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This research project has been funded by the Australian Research Council: DP150103315
The project team thanks Emily Begg for her permission to use her artwork in this article.

Communication in Years 7-9?

Here are some examples of mathematical communication misconceptions which are commonly seen in the classrooms at years 7-9:

- Misuse of the equals sign. Note that misconceptions with this symbol can be quite complex, from the operational role (e.g. 8 + 3 = □) to one of equivalence, (e.g. $2x + 4 = 7 - x$).
  Some common mistakes using the equals sign include:
  - Not using an equals sign between steps, where they should.
  - Using equals signs between ALL steps, regardless of whether this makes sense.
  - Using an implication (⇒) sign instead of equals.
  - Expressing two statements as equal, when they are not.

- Using letters other than familiar ones in algebraic equations and expressions. This can present as:
  - Students not recognising relationships such as Pythagoras' Theorem, when the pronumerals are not the familiar 'a, b or c'.
  - Students not being able to draw graphs, if the variables are not x or y, or if the constant is not labelled 'c' and the gradient 'm'.

- Students not being able to read their mathematics out loud correctly. You might hear:
  - 'x two', instead of x-squared or x to the power of two (x^2), etc.
  - '3 times square root of x', for cube root of x ($\sqrt[3]{x}$).
  - 'Those line things', when referring to symbols, such as the geometry notation ⊥, ∥, ∦, ∡ etc.

Communication in Years 10-12

Here are some examples of mathematical communication problems which you might expect to see in the classrooms at years 10-12:

- Misuse of the equals sign. Note that misconceptions with this symbol can be quite complex, from the operational role (e.g. 8 + 3 = ☐) to one of equivalence, (e.g. $2x + 4 = 7 - x$).
  Some common mistakes using the equals sign include:
  - Not using an equals sign between steps, where they should.
  - Using equals signs between ALL steps, regardless of whether this makes sense.
  - Using an implication (⇒) sign instead of equals.
  - Expressing two statements as equal, when they are not.

- Using letters other than familiar ones in algebraic equations and expressions. This can present as:
  - Students not recognising relationships such as Pythagoras' Theorem, when the pronumerals are not the familiar 'a, b or c'; for example, $sin^2\theta + cos^2\theta = 1$.
  - Students not being able to draw graphs, if the variables are not x or y, or if the constant is not labelled 'c' and the gradient 'm'.
- The meaning of symbols in other classes being ported to mathematics. Some examples could include:
  - x_i could be confused for initial x-value, which is encountered in science.
  - P could be confused for pressure or momentum, rather than probability.
  - i represents current in physics, and is the imaginary number in mathematics.
  - $\log x$ – the assumed base when not explicitly stated can differ, depending upon the subject being studied (IT often assumes base 2, sciences often assume base 10 and mathematics often assumes base e. Some calculators assume base 10 unless explicitly input).

- Difficulty in understanding the use of and/or how to pronounce symbolic notation, such as:
  - Σ (33% of senior secondary and tertiary educators mentioned this symbol).
  - Symbols in set notation, viz. {} | : ∩ etc.
  - Other commonly used Greek alphabet symbols, such as α, β, γ, δ, Δ, ϕ.
  - $f(x)$ read out as 'f of x', but $f(x+h)$ read out as 'f times $x+h$'.
  - $f^{-1}(x)$ as 'one over f of x'.