The essential elements of lessons that include all students in building understanding, solving problems and reasoning mathematically

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Abstract:

- There is widespread agreement that student driven inquiry approaches can assist students in building understanding, solving problems and reasoning mathematically.
- But to ensure that all students are included in the learning opportunities, specific teacher actions are needed and lessons can best be structured in particular ways.
- This presentation uses examples of challenging mathematics learning experiences to illustrate the key elements of such teacher actions and mathematics lessons

What gives you joy when teaching mathematics?

Of course we do not want to initiate or exacerbate anxiety or fear, but which of these two approaches results in a sense of failure and which ends with students feeling they have learned?

- Teach first, moving from simple to complex
- Struggle first, moving from complexity to clarity

Memory is the residue of thought

From Dylan Wiliam

Daniel Willingham, professor of psychology at the University of Virginia.

- Students remember what they have been thinking about, so if you make the learning too easy, students don't have to work hard to make sense of what they are learning and, as a result, forget it quickly.

Note that

- At the end of the day tomorrow, I will illustrate a variety of flexible fluency activities that complement (and are even needed for) what I do now

However you interpret the curriculum or group your students, it is critical that, at least some of the time

- students have a chance to work on tasks prior to being told what to do
- with thoughtful introductions
- and deliberate differentiation
- and structured review of student strategies (where the teaching happens)
- and purposeful follow up

Identifying the key fraction concepts to be learned

4	Investigate equivalent fractions used in contexts (ACMNA077) Count by quarters halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line (ACMNA078)
5	Compare and order common unit fractions and locate and represent them on a number line (ACMNA102) Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator (ACMNA103)

Year 6	Year 7
Compare fractions with related denominators and locate and represent them on a number line (ACMNA125 - Scootle) Solve problems involving addition and subtraction of fractions with the same or related denominators (ACMNA126 - Scootle) Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies(ACMNA127 - Scootle)	Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153 - Scootle) Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154 - Scootle) Recognise and solve problems involving simple ratios(ACMNA173 - Scootle)

Suggestion 1:
Calculating with halves and quarters

- Where the unit is not a whole number
- The emphasis is on imagining buckets and drawing pictures and number lines
- There are various ways of approaching this (this highlights that teaching a single method is limiting) so students have an opportunity to explain their thinking.

As an introduction (that does not “tell”)

- If the purple is worth 1 …
- What is the value of each of the other colours?

How many cups?

The recipe for 4 people uses 2 ½ cups of vegetable stock.

How many cups do I need to make soup for 10 people?

(Work this out two different ways)

Some questions

- What mathematics do students engage with in doing this question?
- What is the point of posing such a question, even if you are not sure that students know enough to engage?
- What might be the risks?

How many cups? (if you are stuck)

- The recipe for 4 people uses 2 cups of vegetable stock.
- How many cups do I need to make soup for 10 people?

How many cups? (if you have finished)

- The recipe for 4 people uses 2 ½ cups of vegetable stock.
- How many cups do I need to make soup for 10 people?
- Work this out a third different way.

How many?

It takes 2 minutes to fill $\frac{3}{4}$ of a bucket.

How many buckets can I fill in 10 minutes?

Show two different ways to find the answer.

How long?

It takes 2 minutes to fill $\frac{3}{4}$ of a bucket.

How long would it take to 9 buckets?

Show two different ways to find the answer.

Some questions

- What mathematics do students engage with in doing those questions?
- What is the point of the set (the sequence) of tasks?
- What might be the risks?

Some feedback from a Year 7 teacher

- They got better at the counting activities
- They persisted
- All made progress so I did not use the prompts
- The projection of answers worked
- I rushed through the first one to see what would happen on the second one
- They worked productively
- All got close to an answer and certainly had a strategy

Suggestion 2: Calculating with thirds

- Where the unit is not a whole number
- The emphasis is on imaging glasses, oranges, etc and drawing pictures and number lines
- There are various ways of approaching this (this highlights that teaching a single method is limiting) so students have an opportunity to explain their thinking.

As an introduction (that does not "tell")

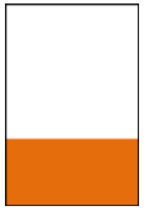
- If the light green is worth 1,
- What is the value of each of the other colours?

How many cups?

It takes 2 minutes to squeeze $\frac{2}{3}$ of a cup of juice.

How many cups of juice could I squeeze in 20 minutes?

Show two different ways to find the answers.

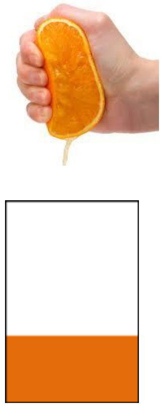


How long?

It takes 2 minutes to squeeze $\frac{2}{3}$ of a cup of juice.

How long would it take to squeeze 8 cups of juice?

Show two different ways to find the answer.



Illustrative enabling prompts

It takes 5 minutes to squeeze 2 cups of juice. How much could I squeeze in 30 minutes?

It takes 5 minutes to squeeze $\frac{1}{2}$ of a cup of juice. How much could I squeeze in 30 minutes?

Illustrative extending prompts

It takes 5 minutes to squeeze $1\frac{2}{3}$ of a cup of juice. How much could I squeeze in $2\,\frac{1}{2}$ hours?

Find a way to represent the answer to any questions like this.

Some questions

- What mathematics do students engage with in doing those questions?
- What is the point of the set (the sequence) of tasks?
- What might be the risks?

Suggestion 3: Changing the representation

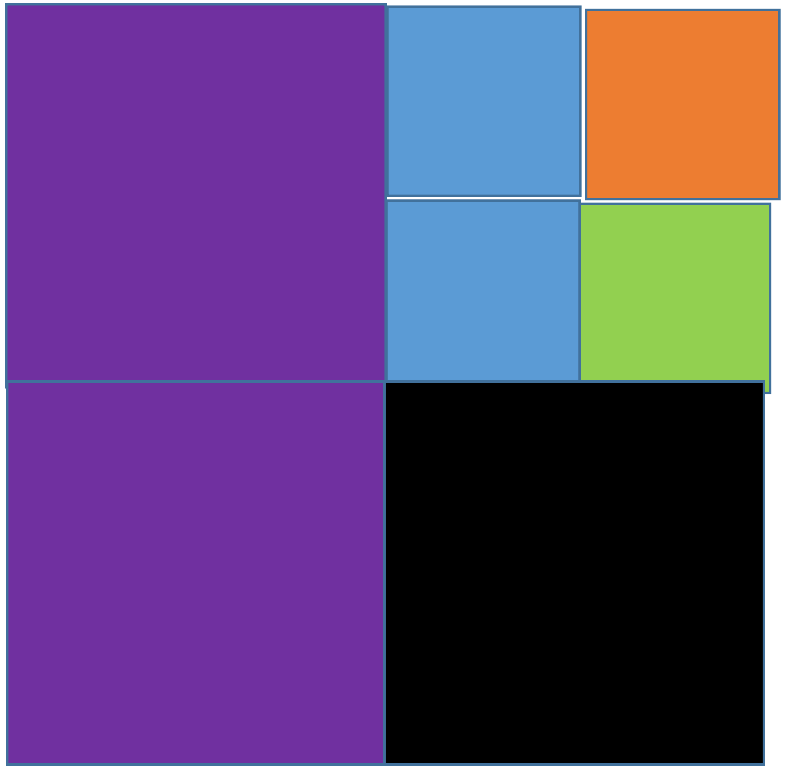
- Changing the context and form of representation
- This time students can use materials to make the “repeated addition” aspect clearer
- Some of these examples require division (or reverse repeated addition)
- The same mathematical strategies can be used even though the representation is different.

The value of the triangle

If each triangle is worth $\frac{2}{5}$ what is the value of the whole shape?

The black is worth $1\frac{1}{3}$

- If the black is worth $1\frac{1}{3}$, what is the value of each of the other colours?



What is the SEQUENCE?

- Is it too much?
- Is it worth it?
- Why would we not just TELL them how to do it?

Not yet	Emerging	Developed	Advanced
	I chose, used and showed some ideas and sometimes connected them together. I used some mathematical words correctly. I used both symbols and drawings..	I chose, used and showed most ideas and mostly connected them together. I used most mathematical words correctly. I connected most symbols and drawings. In most cases, I explained the connection between my two methods.	I chose, used and showed relevant ideas and connected them together. I used mathematical words correctly. I connected symbols and drawings. I explained the connection between my two methods.
	Some of my working out was correct, I multiplied and divided some fractions appropriately and some of my answers were identified.	My working out was mostly complete and correct, The fractions calculations were set out, I multiplied and divided most fractions appropriately and most of my answers were identified.	My working out was complete and correct, I presented calculations efficiently. The fractions calculations were clearly set out, I used the equal sign appropriately, I multiplied and divided fractions appropriately and my answers were clearly identified.
	I explained how I planned and solved some problems, and I checked my solutions.	I explained clearly how I planned and solved most problems, and I checked that my solutions were correct. I used two methods.	I explained clearly how I planned and solved the problems, my methods were creative and I checked that my solutions were correct. My two methods approached the problems in different ways.
	Some steps I took were shown, and I used some examples to show my thinking	The steps I took were mostly shown, and I used some examples to explain and justify my thinking	The steps I took were shown, and I used examples to explain and justify my thinking. I explained how my methods would work

		Advanced
Understanding		I chose, used and showed relevant ideas and connected them together. I used mathematical words correctly. I connected symbols and drawings. I explained the connection between my two methods.
Fluency		My working out was complete and correct, I used appropriate formulas if they were needed, and I presented calculations efficiently, incorporating relevant shortcuts. The fractions calculations were clearly set out, I used the equal sign appropriately, and my answers were clearly identified.
Problem solving		I explained clearly how I planned and solved the problems, my methods were creative and I checked that my solution was correct. My two methods approached the problem in different ways.
Reasoning		The steps I took are shown, and I used examples to explain and justify my thinking. I explained how my methods would work whatever the numbers presented.

However you interpret the curriculum or group your students, it is critical that, at least some of the time

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Some rules

- Experience before instruction
- Give students time
- Do not tell them where the eggs are
- Let students read the question for themselves
- Start at the level that (nearly) all students do not know what to do
- Give them something to talk about
- From active teacher and passive students to …
- Shut up